Printed Pages : 6 **CA-102**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1151 Roll No. | | | | | | | | | | |

M. C. A.

(Semester-I) Theory Examination, 2012-13

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per instructions.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Compute $A(2, 1)$ when $A : N * N \to N$, where N is the set of natural numbers, is defined by :

$$A(0, y) = y + 1$$
$$A(x + 1, 0) = A(x, 1)$$
$$A(x + 1, y + 1) = A(x + 1, y)$$

(b) Find the generating function for the number of r-combinations of $\{3.a, 5.b, 2.c\}$.

(c) Draw the Hasse diagram for the following partial order $(\{1, 2, 3, 4, 5, 6, \leq)$.

(d) Define a complete lattice and give one example.

(e) How many edges does a graph have if it has vertices of degrees 5, 2, 2, 2, 2, 1?

(f) Define homomorphism of a graph. Give suitable example to illustrate.

(g) State pumping lemma for regular language.

(h) Is $[(0+1)-(0b+a1)^*(a+b)]^*$ a regular expression ? Justify your answer.

(i) Differentiate between context-sensitive and context-free grammar using suitable example.

(j) State and explain Halting problem.

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. 10×3=30

2. (a) Solve :

$$S_r - 4S_{r-1} + 4S_{r-2} = 2^r + r2^r.$$

(b) Show that the statement :

$$[p \vee [(\neg r)(\neg s)] \vee [(s((\neg t) \vee p) \vee ((\neg q)r)]$$

is neither a tautology nor a contradiction, where t means truth and p, q, r and s are variables.

(c) Show that in any simple connected planar graph $e \geq 3f/2$ and $e \leq 3n-6$. Here n = number of vertices, e = number of edges and f = number of regions.

(d) Let :

$$V_N = \{S, B\}, V_T = \{a, b\},$$
$$P = \{S \rightarrow aBa, B \rightarrow aBa, B \rightarrow b\}.$$

Find $L(G)$.

(e) Construct a PDA to accept strings containing equal number of a's and b's. Show the moves of the PDA for the input string '*abbaab*'.

Section-C

Attempt *all* questions of this Section. Each question carries 10 marks. 10×5=50

3. Attempt any two parts : 5×2=10

(a) In a survey of 85 people it is found that 31 like to drink milk, 43 like coffee and 39 like tea. Also 13 like both milk and tea, 15 like milk and coffee, 20 like tea and coffee and 12 like none of the three drinks. Find the number of people who like all the three drinks. Use Venn diagram to show your answer.

(b) Which of the following sets are equal ?

$S_1 = \{1, 2, 2, 3\}$, $S_2 = \{x \mid x^2 - 2x + 1 = 0\}$,

$S_3 = \{1, 2, 3\}$, $S_4 = \{x \mid x^3 - 6x^2 + 11x - 6 = 0\}$

(c) Let X is the set of all programs of a given programming language. Let R the relation on X be defined as $P_1 R P_2$ if P_1 and P_2 give the same output on all the inputs for which they terminate. Is R as equivalence relation ?

4. Attempt any one part : 10×1=10

(a) Prove the following :

(i) For any a, b, c, d in a lattice $(A, \leq)$, if $a \leq b$ and $c \leq d$ then :

$(a \vee c) \leq (b \vee d)$ and $(a \wedge c) \leq (b \wedge d)$

(where $\vee$ is join and $\wedge$ is meet operation).

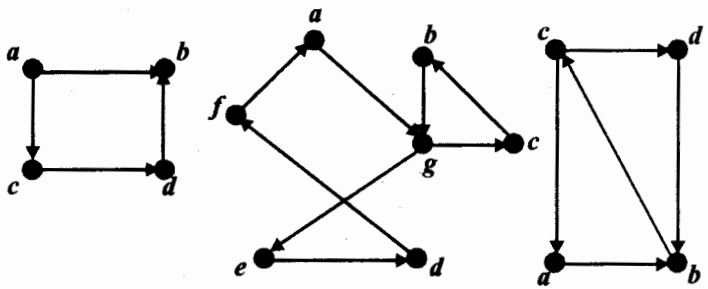
(ii) If the meet operation is distributive over the join operation in a lattice, then the join operation is also distributive over the meet operation.

(b) What is an abelian group ? Let (G, o) be a group. Show that (G, o) is an abelian group if and only if $(aob)^2 = a^2ob^2$, for all a, b in G.

5. Attempt any two parts : $5\times2=10$

(a) What do you mean by a planar graph ? Show that the graph K_5 is not planar.

(b) Which of the following graphs H_1, H_2 and H_3, have an Euler circuit ? Identify the one that is not Euler circuit but an Euler path. Justify.



H_1 H_2 H_3

(c) Make a Binary Search Tree (BST) for the following sequence of numbers :

45, 36, 76, 23, 89, 115, 98, 39, 41,56, 69, 48

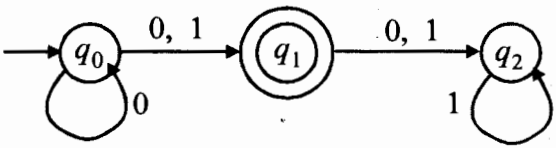
Traverse the tree in preorder, inorder and postorder.

6. Attempt any two parts : 5×2=10

(a) Show that complement of a regular set is a regular set.

(b) Design a Mealy machine that prints 1's complement of an input bit string.

(c) Convert the non-deterministic finite automata in the following graph into equivalent deterministic machine :



7. Attempt any one part : 10×1=10

(a) Design a Turing Machine that recognizes the language consisting of all strings of even length over alphabet $\{a, b\}$.

(b) Define the following languages. Also show pictorially the relationship between them :

(i) Recursively enumerable

(ii) Recursive, and

(iii) Non-recursively enumerable.